Combining general relativity and quantum theory: points of conflict and contact

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Abstract. The issues related to bringing together the principles of general relativity and quantum theory are discussed. After briefly summarising the points of conflict between the two formalisms I focus on four specific themes in which some contact has been established in the past between GR and quantum field theory: (i) The role of planck length in the microstructure of spacetime (ii) The role of quantum effects in cosmology and origin of the universe (iii) The thermodynamics of spacetimes with horizons and especially the concept of entropy related to spacetime geometry (iv) The problem of the cosmological constant.

1. Introduction

The question of bringing together the principles of quantum theory and gravity deserves to be called *the* problem of theoretical physics today. In this review I shall highlight the points of conflict *and* contact between these two theoretical structures focusing on four major themes which run through all the work in quantum gravity over decades: (i) What is the role played by the length scale $L_P \equiv (G\hbar/c^3)^{1/2}$ in determining the spacetime microstructure (see eg. [1] - [5]) ? (ii) What have we learnt regarding the role of quantum gravity in quantum cosmology and in the origin of the universe (see eg.[6] - [12])? (iii) To what extent do we understand the thermodynamics of spacetimes with horizons (see eg. [13] - [18]) ? (iv) What is the role of quantum gravity vis-a-vis the cosmological constant (see eg. [19],[20]) ? Even among these themes, I will concentrate more on the latter two. Before I discuss the concrete issues, it is probably worth comparing some general aspects of quantum theory and general relativity.

2. The miracle of quantum field theory

The key feature of quantum field theory is that *it has no right to be as successful as it is!*. In proceeding from classical mechanics [with finite number of degrees of freedom] to quantum mechanics, one attributes operator status to various dynamical variables and imposes the commutation relations among them. Often, it is convenient to provide a representation for the operators in terms of normal differential operators so that the problem can be mapped to solving a partial differential equation — say, the time-dependent Schroedinger equation — with specific boundary conditions. Such

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problems are mathematically well defined and tractable, allowing us to construct a well defined [though, in general, not unique] quantum theory for a classical system with finite number of degrees of freedom.

The generalisation of such a procedure to a *field* with infinite number of degrees of freedom is *not* straightforward. Given a classical field with some dynamical variables, one can attempt to quantise the system by elevating the status of dynamical variables to operators and imposing the commutation rules. But finding a well defined and meaningful representation for this commutator algebra is a nontrivial task. Further, if one tries to extend the approach of quantum mechanics [based on Schroedinger picture] to the field, one obtains a *functional* differential equation instead of a partial differential equation. The properties — let alone solutions! — of this equation are not well understood for any field with nontrivial interactions. Somewhat simpler (and better) approach will be to use the Heisenberg picture and try to solve for the operator valued distributions representing the various dynamical variables. Even in this case, one does not have a systematic mathematical machinery to solve these equations for an interacting field theory. The evolution equations for operators in QED [in 3+1 dimensions], for example, cannot be solved exactly; however, it is possible to set up a perturbation expansion for the relevant variables in powers of the coupling constant $(e^2/\hbar c) \approx 10^{-2}$. The lowest order of the perturbation series, in which all interactions are switched off, defines the *free field* theory which can be mapped to a model describing infinite number of noninteracting harmonic oscillators. The perturbation expansion can be then used to obtain the "corrections" to this free field theory. Several nontrivial conceptual issues crop up when such an attempt is made:

(a) To begin with, the decomposition of the field in terms of the harmonic oscillators is not unique and there exists infinite number of inequivalent representations of the basic commutator algebra for the system. This shows that "physical" quantities like ground state, particle number etc. will depend on the specific representation chosen and will not be unique.

(b) Since the system has infinite number of degrees of freedom, quantities like total energy can diverge. The actual form of the divergence depends on the representation chosen for the algebra and the differences between infinite quantities may retain a representation dependent [finite] value, unless one is careful in regularising such expressions. In some cases, one may be forced to choose particular set of harmonic oscillators because of the boundary conditions. Then, the difference between two infinite quantities could be physically relevant (and even observable as in the case of, for example, Casimir effect).

(c) The situation becomes worse when the perturbation is switched on. In general, the perturbation series will not converge and has to be interpreted as an asymptotic expansion. Further, the individual terms in the perturbation series will not, in general, be finite creating a far more serious problem. This is related to the fact that virtual quanta of *arbitrarily* high energy are allowed to exist in the theory [needed for incorporating Lorentz invariance at arbitrarily small length scales] and still propagate as free fields.

(d) Perturbation theory completely misses all effects which are nonanalytic in the coupling constant. In QED, for example, perturbation theory cannot lead to the result that an external electromagnetic field can produce $e^+ - e^-$ pairs [21] since this effect has nonanalytic dependency on e through a factor $\exp\left[-\left(\pi m_e^2/|eE|\right)\right]$.

How does one cope up with these difficulties? Issue (a) is handled by choosing

one particular representation for the free field theory by fiat, and working with it — and ignoring all other representations. This also dodges the issue (b) through normal ordering, once a representation for the harmonic oscillators is chosen. Issue (d) is accepted as a failure of the method and then ignored. Most of the successful effort was concentrated on handling the problem of infinities *in the individual terms* of the perturbation series, that is, on issue (c). The paradigm for handling these infinities can be stated in terms of the concept of renomalization which – though has nothing to do with divergences, a priori – does allow one to cure the divergences, if *all* the divergent terms of a perturbation expansion can be eliminated by redefining the coupling constants in the theory. For an arbitrary field theory, we have no assurance that all the divergences can be so eliminated; in fact, it is quite easy to construct well defined classical field theories for which divergences cannot be eliminated by this process.

The unexplained miracle of quantum field theory lies in the fact that several physically relevant field theories — describing quantum electrodynamics, electro-weak interactions and QCD — belong to this special class of *perturbatively renormalisable* theories. Nobody knows why this mathematically non-rigorous, conceptually ill-defined, formalism of perturbative quantum field theory works. The miracle becomes even more curious when we notice that the bag of tricks fails miserably in the case of gravity.

3. Gravity: Thorn in the flesh

Until seventies, most of the hardcore particle physicists used to ignore general relativity and gravitation and the first concrete attempts in putting together principles of quantum theory and gravity were led by general relativists (see e.g. [6]). It was clear, right from the beginning, that this is going to be a formidable task since the two "theories of principle" differed drastically in many aspects:

(a) The Lagrangian describing classical gravity, treated as a function of $h_{ik} = g_{ik} - \eta_{ik}$, is *not* perturbatively renormalizable; in fact, there does not exist any simple redefinition of the field variables which will lead to a perturbatively renormalizable theory. So the most straight forward approach, based on the belief that nature will continue to be kind to us, is blocked. The miracle fails.

(b) The principle of equivalence implies that any reasonable description of gravity will have a geometrical structure and that gravitational field will affect the spacetime intervals in a specific manner, thereby making the spacetime itself dynamical. For a general gravitational field, there will be no way of choosing a special class of spacelike hypersurfaces or a time coordinate.

(c) Gravity affects the light signals and hence determines the causal structure of spacetime. In particular, gravity is capable of generating regions of spacetime from which no information can reach the outside world through classical propagation of signals. This feature, which may be loosely called 'the existence of trapped surfaces' has no parallel in any other interaction. When gravity makes certain regions inaccessible, the data regarding quantum fields in these regions can "get lost". This requires reformulation of the equations of quantum field theory, possibly by tracing over the information which resides in the inaccessible regions — something which is not easy to do either mathematically or conceptually.

(d) Since all matter gravitates, the gravitational field becomes more and more dominant at larger and larger scales. In the limit, the asymptotic structure of

spacetime is determined by global, smoothed out distribution of matter in the cosmological context. Hence, the spacetime will not be asymptotically flat in the spatial variables at any given time. The behaviour of the spacetime for $t \to \pm\infty$ will also be highly non-trivial and could be dominated by very strong gravitational fields.

(e) All energies gravitate thereby removing the ambiguity in the zero level for the energy, which exists in non-gravitational interactions. This feature also suggests that there is no such thing as a free, non-interacting field. Any non trivial classical field configuration will possess certain amount of energy which will curve the spacetime, thereby coupling the field to itself indirectly. Gravitational field is not only nonlinear in its own coupling, but also makes *all matter fields* self-interacting.

These features create problems even when one tries to develop a quantum field theory in an external gravitational field. Conventional quantum field theory works best when a static causal structure, global Lorentz frame, asymptotic in-out states, bounded Hamiltonians and the language of vacuum state, particle excitations etc., are supplied. The gravitational field removes all these features, strongly hinting that we may be working with an inadequate language. Perturbative language which — at best — gives an algorithm to calculate S-matrix elements, is not going to be of much use in understanding the quantum structure of gravitational field. Most of the interesting questions — possibly *all* the interesting questions — in quantum gravity are non perturbative in character; whether a theory is perturbatively renormalizable or not is totally irrelevant in this context. The gradual paradigm shift in the particle physics community from perturbative renormalisability (in 70's) through perturbative finiteness of supergravity (in early 80's) to non perturbative description of superstrings (in late 90's) represents a grudging acceptance of the lessons from gravity.

Finally, one may ask — given these difficulties — is it really necessary to quantise gravity ? The answer is "yes" and can be proved in two steps: (i) One can easily prove that if the Casimir energy does not gravitate, it is possible to construct a perpetual motion machine using two Casimir plates and a set of weights and pulleys (see e.g [22]). (ii) If the source for gravity is quantum mechanical (like Casimir energy) but the field is classical then it is possible to violate the uncertainty principle by a suitable set up. Thus at least some minimal amount of quantum structure need to be imposed on gravity.

4. Role of Planck length in the microstructure of spacetime

Having summarised the points of conflict, let me now turn to the points of contact, beginning with the first of the four themes I mentioned in the Introduction. The fact that all matter gravitates stresses the need to abandon description based on free field theory to handle virtual excitations with arbitrarily high energies. An excitation with energy E will probe length scales of the order of $(1/E)$ and when $E \to E_P$, the nonlinearity due to self gravity cannot be ignored for any field. The same conclusion is applicable even to vacuum fluctuations of any field, including gravity. If we attempt to treat the ground state of the gravitational field as the flat spacetime, we must conclude that the spacetime structure at $L \lesssim L_P$ will be dominated by quantum fluctuations of gravity and the smooth macroscopic spacetime can only emerge when the fluctuations are averaged over larger length scales. Hence the description of continuum spacetime in terms of, classical, Einstein's equation should be thought of as similar to the description of a solid by elastic constants. While the knowledge of microscopic quantum theory of atoms and molecules will allow us, in principle, to

construct the description in terms of elastic constants, the reverse process is unlikely to be unique. What one could hope is to take clues from well designed thought experiments, thereby identifying some key generic features of the microscopic theory.

To begin with, one can prove – using well-chosen thought experiments – that it is not possible to measure intervals smaller than $L_P = (G\hbar/c^3)^{1/2}$. (This is demonstrated in [23]; a clear statement in this direction, based on a toy model, is in [2] and a more "modern" approach to the same result is in [24].) More formally, one can prove that the quantum fluctuations in the metric lead to the following limit (see ref.[23], [25].)

$$\lim_{x\to y} < l^2(x,y) > \approx (x-y)^2 + L_P^2 \tag{1}$$

where l is the geodesic distance between x^i and y^i and the averaging is over all metric fluctuations. This suggests that Planck length should be thought of as the "zero-point length" of the spacetime and any correct theory of quantum gravity must incorporate this feature in a suitable form.

One specific consequence of this result is in the case of a Friedmann model for the universe. It can be shown that [26] the lower bound at Planck length leads to an effective metric of the form

$$ds^2 = dt^2 - L_P^2(n+\frac{1}{2})\left[d\chi^2 + \sin^2\chi(d\Omega^2)\right] \tag{2}$$

leading to the *areas* of spherical surfaces being quantised in units of L_P^2.

It is *not* possible to reconcile the the existence of a zero point length in (1) with a Lorentz invariant, local, QFT description. Both string theory and loop gravity (the two approaches which have been worked out to fair degree of detail) incorporate this lower bound in different ways. In string theory, nonlocality is built in and hence it is possible to obtain Lorentz invariance at the long-wavelength limit. The situation is less clear in loop gravity but it has the most direct implementation of this principle in, for example, area quantisation. In loop gravity the area operator is quantised but the eigenvalues scale differently [3] compared to (2). More recently, there has been attempts to construct quantum cosmological models based on loop gravity (see e.g. [12]) and it appears that results like (2) might arise as an asymptotic limit.

I have attempted [5] to use the interpretation of L_P as a zero-point length to provide a working description of quantum field theory which is free of ultraviolet divergences. The starting point of this analysis is to modify the path integral for Euclidean Green function $G_F(x,y)$ in such a way that the path integral amplitude is invariant under the "duality" transformation $l \to (L_P^2/l)$ where $l^2 = (x-y)^2$. This demands the replacement

$$G(x,y) = \sum e^{-l(x,y)} \to G_{\rm modified} \equiv \sum \exp[-(l+\frac{L_P^2}{l})] \tag{3}$$

Remarkably enough, it turns out that the path integral sum in (3) can be evaluated rigorously. The final result is quite simple: the modified Green function is related to the original one by the replacement $(x-y)^2 \to [(x-y)^2 + L_P^2]$! That is, $G_{\rm modified}(x,y) = G_{\rm usual}[l^2 \to (l^2 + L_P^2)]$ In other words, the postulate of duality (as defined above) implies the existence of a zero-point length. It is known that string theories – which have zero-point length built in – do lead to dualities of different kinds. I would like to stress that there is no simple reason to expect, a priori, a connection between (3) and zero-point length.

Once the postulate of path integral duality is accepted, it is possible to obtain several interesting consequences: (i) To begin with, it is clear that gravity acts as a *non perturbative* regulator. For example, the modified Feynmann propagator for a massless scalar field has the structure (see [2],[5]):

$$\frac{1}{x^2} \to \frac{1}{(x^2 + L_P^2)} = \frac{1}{x^2} - \frac{L_P^2}{x^4} + \cdots \tag{4}$$

Each term on the right hand side diverges as $x \to 0$ and only the sum remains finite in the coincidence limit, as $x \to 0$. (ii) One can compute the corrections to the bare cosmological constant $\Lambda_{\rm bare}$ and Newtonian gravitational constant $G_{\rm bare}$ when other fields are integrated out in a path integral. One finds that

$$\Lambda_{\rm ren} = \Lambda_{\rm bare} - \frac{1}{4\pi\eta^4 G}; \quad G_{\rm ren}^{-1} = G_{\rm bare}^{-1}\left[1 + \frac{1}{12\pi\eta^2}\right] \tag{5}$$

where η is a pure number related to the number of scalar fields integrated out. The result shows that the value $\Lambda = 0$ *is unprotected* against large quantum gravitational corrections. (iii) Any form of area quantisation implies that the density of BH states on the horizon is of the order of (A/L_P^2) with clear implications for the entropy of blackhole. (iv) The zero point length also suggests that there will be exponential suppression of modes shorter than Planck length. This, for example, will allow inflation at quantum gravitational scales [27] since the production of gravitational waves will be suppressed.

5. Why did the universe become classical ?

Let me now turn to the second theme, viz. quantum cosmology. Considering the fact that quantum cosmology was one of the earliest points of contact between QT and GR, it is rather disappointing that it has not produced any concrete results. Fundamental questions regarding the origin of the universe remain unanswered in all approaches and even the descriptive language requires semiclassical crutches. Serious technical questions (eg, exact validity of minisuperspace, canonical vs path integral approaches, Euclidean vs Lorentzian path integral, topology change .....) are still controversial.

Most of the early work in quantum cosmology was on the question of singularities (and "creation" of the universe) and these models (see eg.[7], [8], [26], [28]) were the precursors of the currently more fashionable [though hardly better justified] pre-bigbang models. It is fairly easy to construct toy quantum cosmological models without singularity or horizon. One example I worked out long back [28] has the effective metric:

$$ds^2 \cong \left(\alpha L_P^2 + \tau^2\right)\left[d\tau^2 - dx^2 - dy^2 - dz^2\right]; \quad \alpha = \mathcal{O}(1) \tag{6}$$

The difficulty with such pre-big bang models is that they are hopelessly diverse and do not give any more insight than the original assumptions of the model.

Somewhat more concrete results exist as regards the semiclassical limit of the quantum cosmology, especially since we cannot invoke an "observer' in this context. It is possible to show that decoherence provides an answer and leads to the density matrix of 3-geometries becoming effectively diagonal. One can define a "distance" in the superspace of 3-geometries $l^2(g_{\alpha\beta},\ g'_{\alpha\beta})$ in terms of which one can illustrate [9] the suppression of off-diagonal components of the density matrix explicitly:

$$\rho_{\rm off-diagonal} \approx \rho_{\rm diagonal}\ \exp\left[-\frac{l^2(g_{\alpha\beta},\ g'_{\alpha\beta})}{L_P^2}\right] \tag{7}$$

There is another interesting aspect [30] to this analysis: Among all systems dominated by gravity, the universe possess a very peculiar feature. If the conventional cosmological models are reasonable, it then follows that *our universe proceeded from quantum mechanical behaviour to classical behaviour in the course of dynamical evolution defined by some intrinsic time variable.* In terms of Wigner functional $W(g,p)$, this transition can be stated as evolution leading to,

$$W(g_{\alpha\beta}, p^{\alpha\beta}) = A(g)B(p) \quad \rightarrow \quad W(g_{\alpha\beta,} p^{\alpha\beta}) = F\left[p - p_{\rm class}(g)\right] \quad (8)$$

where A, B are arbitrary functionals and F is a functional sharply peaked on its argument. This transition is *not* possible for systems with bounded Hamiltonians arising in a low-energy effective theory with finite number of fields integrated out. It follows that the quantum cosmological description of our universe, as a Hamiltonian system, should contain at least one unbounded degree of freedom. In simple quantum cosmological models, one can write $\mathcal{H} = \mathcal{H}_{\rm unbound}(a) + \mathcal{H}_{\rm bound}(a,q)$; where a is the expansion factor and q denotes all other degrees of freedom. It can also be shown that the unbounded mode — which, in the case of FRW universe, corresponds to the expansion factor $a(t)$ — will become classical first, as is experienced in the evolution of the universe.

One might assume that the microscopic description of spacetime is in terms of certain [as yet unknown] variables q_i and that the conventional spacetime metric is obtained from these variables in some suitable limit. Such a process will necessarily involve coarse-graining over a class of microscopic descriptors of geometry. If one starts with a bounded Hamiltonian for a system with *finite* number of quantum fields and integrate out a subset of them, the resulting Hamiltonian for the low energy theory cannot be unbounded. Assuming that the original theory is describable in terms of a bounded Hamiltonian for some suitable variables, it follows that an infinite number of fields have to be involved in its description and an infinite subset of them have to be integrated out in order to give the standard low energy gravity. This feature is indeed present in one form or the other in the descriptions of quantum gravity based on strings or loop variables.

These arguments can be extended further. Starting from an (unknown) quantum gravitational model, one can invoke a sequence of approximations to progressively arrive at quantum field theory (QFT) in curved spacetime, QFT in flat spacetime, nonrelativistic quantum mechanics and Newtonian mechanics. The more exact theory can put restrictions on the range of possibilities allowed for the approximate theory which are *not derivable from the latter* – an example being the symmetry restrictions on the wave function for a pair of electrons in point QM which has its origin in QFT. The choice of vacuum state at low energies could be such a "relic" arising from combining the principles of quantum theory and general relativity [29]. The detailed analysis suggests that the wave function of the universe, when describing the large volume limit of the universe, dynamically selects a vacuum state for matter fields — which, in turn, defines the concept of particle in the low energy limit. The result also has the potential for providing a concrete quantum mechanical version of Mach's principle.

6. Thermodynamics and/of geometry: Can cosmological constant evaporate?

One of the remarkable features of classical gravity is that it can wrap up regions of spacetime thereby producing surfaces which act as one way membranes. The classic example is that of a Schwarzschild blackhole which has a compact surface acting as observer independent event horizon. Another example is the deSitter universe which also has an one way membrane; but the location of the horizon depends on the observer and hence is coordinate dependent. In fact, the existence of one-way membranes is not necessarily a feature of curved spacetime; it is possible to introduce coordinate charts even in the flat Minkowski spacetime, such that regions are separated by horizons. The familiar example is the Rindler coordinate frame which has a non-compact surface acting as a coordinate dependent horizon.

All the three spacetimes mentioned above (Schwarzschild, deSitter, Rindler) as well as a host of other spacetimes with horizons can be described in a general manner as follows. Consider a $(D+1)$-dimensional *flat* Lorentzian manifold with the line element

$$ds^2 = (dZ^0)^2 - (dZ^1)^2 ....(dZ^D)^2 \equiv \eta_{AB} dZ^A dZ^B \tag{9}$$

Spacetimes of relevance to us can all be thought of $4-$dimensional sub manifolds of this $(D+1)-$dimensional manifold, defined in a suitable manner. I will first introduce two new coordinates (t, r) in place of (Z^0, Z^1) through the definitions

$$Z^0 = l f(r)^{1/2} \sinh \quad gt; \quad Z^1 = \pm l f(r)^{1/2} \cosh gt \tag{10}$$

where (l, g) are constants introduced for dimensional reasons and we will usually take $l \propto (1/g)$. Clearly the pair of points (Z^0, Z^1) and $(-Z^0, -Z^1)$ are mapped to the same (t, r) making this a 2-to-1 mapping. (The transformations in (2) covers only the two quadrants with $|Z^1| > |Z^0|$ with the positive sign for the right quadrant and negative sign for the left but can easily be extended to other quadrants with sinh and cosh interchanged). Note that if one introduces the Euclidean continuation of the time coordinates with $iZ^0 \equiv T; it = \tau$, the transformations in (2) continue to be valid but with a periodicity of $(2\pi/g)$ in τ. With the transformation in (2), the metric in (9) becomes

$$ds^2 = f(r)(lg)^2 dt^2 - \frac{l^2}{4}\left(\frac{f'^2}{f}\right) dr^2 - dL^2_{D-1} \tag{11}$$

in all the four quadrants. The choice of D and the definition of the four dimensional subspace depends on the spacetime we are interested in: (a) In the simplest case of Rindler spacetime we can take $f = (1 + 2gr), l = g^{-1}$ with all the (D-2) transverse dimensions going for a ride. In fact, we can treat this case as just a redefinition of coordinates, involving a mapping from $(D + 1) = 4$ to $(D + 1) = 4$. (b) If we take $(D + 1) = 5$ and use — in addition to the mapping given by (2) — a transformation of Cartesian (Z^2, Z^3, Z^4) to the standard spherical polar coordinates: $(Z^2, Z^3, Z^4) \to (r, \theta, \varphi)$, and choose $lg = 1; f(r) = \left[1 - (r^2/l^2)\right]$, we get the deSitter spacetime in static coordinates. (c) To obtain the Schwarzschild spacetime‡ we start with a $(D + 1) = 6$-dimensional spacetime $(Z^0, Z^1, Z^2, Z^3, Z^4, Z^5)$ and consider a mapping to 4-dimensional subspace in which: (i) The (Z^0, Z^1) are mapped to (t, r)

‡ This was first obtained in [33] but the analysis in this reference hides the simplicity and generality of the result.

as before; (ii) the (Z^3, Z^4, Z^5) are mapped to standard spherical polar coordinates: (r, θ, φ) *and* (iii) we take Z^2 to be an arbitrary function of r: $Z^2 = q(r)$. This leads to the metric

$$ds^2 = A(r)dt^2 - B(r)dr^2 - r^2 d\Omega^2_{2-\rm sphere}; \tag{12}$$

with

$$A(r) = (lg)^2 f; \qquad B(r) = 1 + q'^2 + \frac{l^2}{4}\frac{f'^2}{f} \tag{13}$$

This choice will allow us to obtain *any* spherically symmetric, static, 4-dimensional spacetime. For the Schwarzschild solution we will take $2lg = 1, f = 4\left[1 - (l/r)\right]$; and

$$q(r) = \int^r \left[\left(\frac{l}{r}\right)^3 + \left(\frac{l}{r}\right)^2 + \frac{l}{r}\right]^{1/2} dr \tag{14}$$

Though the integral cannot be expressed in terms of elementary functions, it is obvious that $q(r)$ is well behaved everywhere including at $r = l$.

In the examples of spacetimes with horizons, $f(r)$ vanishes at some $r = l$ so that $g_{00} \approx |(r/l - 1)|$ near $r = l$; such spacetimes have a horizon at $r = l$. There exists a natural definition of QFT in the original (D + 1)-dimensional space and we can define a vacuum state for the quantum field on the $Z^0 = 0$ surface, which coincides with the $t = 0$ surface. It is straightforward to show that this vacuum state appears as a thermal state with temperature $T = (g/2\pi)$ in the 4-dimensional subspace. The most important conclusion which follows from this analysis [31] is that the existence of the temperature is a purely kinematic effect arising from the coordinate system we have used — which should also be obvious from the fact that (10) implies periodicity in imaginary time coordinate.

The QFT based on such a state will be manifestedly time symmetric and will describe an isolated system in thermal equilibrium in the subregion $\mathcal{R}$. No time asymmetric phenomena like evaporation, outgoing radiation, irreversible changes etc can take place in this situation. This is gratifying since one may be hard pressed to interpret an evaporating Minkowski spacetime or even an observer dependent evaporation of a deSitter spacetime; by choosing to work with the quantum state which is time symmetric we can bypass such conceptual issues.

It is also possible to show that 4-dim QFT gets mapped to 2-dim Conformal Field Theory (CFT) in all these spacetimes. Consider, for example, a QFT for a self-interacting scalar field in a spacetime with the metric of the form in equation (11):

$$ds^2 = f(r)(lg)^2 dt^2 - \frac{l^2}{4}\left(\frac{f'^2}{f}\right) dr^2 - g_{\alpha\beta}dx^\alpha dx^\beta; \quad g_{\alpha\beta} = g_{\alpha\beta}(r, \mathbf{x}_\perp) \tag{15}$$

where the line element $g_{\alpha\beta}dx^\alpha dx^\beta$ denotes the irrelevant transverse part corresponding to the transverse coordinates $\mathbf{x}_\perp$, *as well as* any regular part of the metric corresponding to dr^2. The field equation for a scalar field in this metric can be reduced to the form

$$\frac{\partial^2\phi}{\partial t^2} - \frac{\partial^2\phi}{\partial\xi^2} = \left(\frac{2g}{f'}\right)^2 \frac{f^2}{Q}\left(\frac{\partial\phi}{\partial r}\right)\left(\frac{\partial Q}{\partial r}\right) + (lg)^2 f\left[(\nabla^2_\perp\phi) + \frac{\partial V}{\partial\phi}\right] \tag{16}$$

where $2g\xi = lnf$. The right hand side vanishes as $r \to l$ because f vanishes faster than all other terms. It follows that near the horizon we are dealing with a (1+1) dimensional CFT governed by

$$\frac{\partial^2\phi}{\partial t^2} - \frac{\partial^2\phi}{\partial\xi^2} \approx 0 \tag{17}$$

which has an extra symmetry of conformal invariance.

If we take $f(r) \propto (r - l)$ near $r = l$ and separate the time dependence by $\phi = \phi_\omega e^{-i\omega t}$, it is easy to see that – near $r = l$ – the fundamental wave modes are $\phi = e^{-i\omega t \pm i\omega\xi} = e^{-i\omega(t\pm\xi)} = \left(e^{-i\omega z}, e^{-i\omega\bar{z}}\right)$ where $\tau = it$ and $z \equiv (\xi + i\tau)$ is the standard complex coordinate of the conformal field theory. The boundary condition on the horizon can be expressed most naturally in terms of z and $\bar{z}$. For example, purely ingoing modes are characterised by $(\partial f/\partial\bar{z}) = 0; (\partial f/\partial z) \neq 0$. Since the system is periodic in τ, the coordinate z is on a cylinder ($R^1 \times S^1$) with τ being the angular coordinate (S^1) and ξ being the R^1 coordinate. The periodicity in τ is clearer if we introduce the related complex variable ρ by the definition $\rho = \exp g(\xi + i\tau) = \exp(gz)$. The coordinate ρ respects the periodicity in τ and is essentially a mapping from a cylinder to a plane. It follows that the modes $(e^{-i\omega z}, e^{-i\omega\bar{z}})$ become $(\rho^{-i\omega/g}, \bar{\rho}^{-i\omega/g})$ in terms of ρ.

The situation is simpler in the case of a free field with $V = 0$. Then one can show that, near the horizon, $r \simeq l, r' \simeq l$ the two point function will have the limiting form

$$G\left(t - t'; r, r'; \mathbf{x}_\perp, \mathbf{x}'_\perp\right) \cong \left\{\sum_\lambda f_\lambda(\mathbf{x}_\perp) f_\lambda(\mathbf{x}'_\perp)\right\}\left\{\sum_\omega e^{-i\omega[(t-t')\pm(\xi-\xi')]}\right\} \tag{18}$$

where the function f is the eigenfunction of transverse Laplacian with (set of) eigenvalue(s) λ; that is $\nabla^2_{D-1} f = -\lambda^2 f$. In other words, the two point function factorises into a transverse and radial part with the radial part being that of a two dimensional massless scalar field. The latter is the same as the Green function of the standard conformal field theory.

The role of horizon in producing a CFT can be summarised as follows: In a general $(D + 1)$ dimensional theory with $D \geq 2$, we do not have conformal invariance. If we can kill the transverse dimensions and reduce the theory to a 2-dimensional theory, then we would have automatically enhanced the symmetries to that of a CFT. The metrics in (11) with g_{00} having a simple zero at $r = l$ achieves exactly this. Since such metrics have a horizon, we obtain a connection between CFT and horizons quite generically. All the results of CFT (especially the behaviour of two-point functions) can now be used to study the field theory near the horizon.

Trapped surfaces also highlight the role of boundary conditions (called holographic principle in some contexts) in QFT. The structure of a free field propagating in an arbitrary spacetime can be completely specified in terms of, say, the Feynmann Greens function $G_F(x, y)$ which satisfies a local, hyperbolic, inhomogeneous, partial differential equation. Each solution to this equation provides a particular realization of the theory so that there exists a mapping between the realizations of the quantum field theory and the relevant boundary conditions to this equation which specify a useful solution. When trapped surfaces exists, the differential operator governing the Greens function will be singular on these surfaces (in some coordinate chart) and the issue of boundary conditions become far more complex. It is, nevertheless possible — at least in simple cases with compact trapped surfaces — to provide an one-to-one correspondence between the ground states of the theory and

the boundary conditions for G_F on the compact trapped surface. In fact, the Greens function connecting events outside the trapped surface can be completely determined in terms of a suitable boundary condition on the trapped surface, indicating that trapped surfaces acquire a life of their own even in the context of QFT in CST. In a way, the procedure is reminiscent of renormalisation group approach, but now used in real space to integrate out information inside the trapped surface and replace it by some suitable boundary condition.

One would next like to know whether one can associate an entropy with these spacetimes in a sensible manner, given that the notion of temperature arises very naturally. Conventionally there are two *very different* ways of defining the entropy. In statistical mechanics, the entropy $S(E)$ is related to the degrees of freedom [or phase volume] $g(E)$ by $S(E) = \ln g(E)$. Maximisation of the phase volume for systems which can exchange energy will then lead to the equality of the quantity $T(E) \equiv (\partial S/\partial E)^{-1}$ for the systems. It is conventional to identify this variable as the thermodynamic temperature. In classical thermodynamics, on the other hand, it is the *change in* the entropy, which can be operationally defined via $dS = dE/T(E)$. Integrating this equation will lead to the function $S(E)$ except for an additive constant which needs to be determined from additional considerations.

In the case of time symmetric spacetimes, if one chooses a vacuum state of QFT which is also time symmetric, then there will be no change of entropy dS and the thermodynamic route is blocked. But the alternative definition of S — in terms of certain degrees of freedom — is possible even in the time symmetric context. Unfortunately, identifying these degrees of freedom is a nontrivial task. More importantly, the QFT described in the last few paragraphs makes absolutely no mathematical distinction between the horizons which arise in the Schwarzschild, deSitter and Rindler spacetimes. Any honest identification of degrees of freedom in conventional QFT will lead to a definition of entropy *for all the three cases.* While the blackhole result is acceptable (the horizon being compact and observer independent), the deSitter spacetime will have an observer dependent entropy (the horizon is compact but coordinate dependent) and the Rindler frame will have an infinite, observer dependent entropy (since the horizon is non-compact and observer dependent). While there is voluminous literature on the temperature associated with these spacetimes, there is virtually no clear, published, discussion on the question: *Does the deSitter and Rindler spacetime possess observer dependent entropies, which can be interpreted sensibly?*

There is an alternative point of view which one can take regarding this issue. The Schwarzschild metric, for example, can be thought of as an asymptotic metric arising from the collapse of a body forming a blackhole. While developing the QFT in such a spacetime we need not maintain time reversal invariance for the vacuum state and — in fact — it is more natural to choose a state with purely ingoing modes at early times like the Unruh vacuum state. The study of the QFT in such a spacetime shows that, at late times, there will exist a thermal, outgoing, radiation of particles which is totally independent of the details of the collapse. The temperature in this case will be $T(M) = 1/8\pi M$, which is the same as the one found in the case of the state of thermal equilibrium around an "eternal" blackhole. In the Schwarzschild spacetime, which is asymptotically flat, it is also possible to associate an energy $E = M$ with the blackhole. Though the QFT in CST calculation was done in a metric with fixed value of energy $E = M$, it seems reasonable to assume that as the energy flows to infinity at late times, the mass of the black hole will decrease. *If* we make this assumption

— that the evaporation of black hole will lead to decrease of M — *then* one can integrate the equation $dS = dM/T(M)$ to obtain the entropy of the blackhole to be $S = 4\pi M^2 = (1/4)(A/L_P^2)$ where $A = 4\pi(2M)^2$ is the area of the event horizon and $L_P = (G\hbar/c^3)^{1/2}$ is the Planck length. [This integration can determine the entropy only upto an additive constant. To fix this constant, one can make the additional assumption that S should vanish when $M = 0$. One may think that this assumption is eminently reasonable since the Schwarzschild metric reduces to Lorentzian metric when $M \to 0$. But note that in the same limit of $M \to 0$, the temperature of the blackhole diverges !. Treated as a limit of Schwarzschild spacetime, normal flat spacetime has infinite — rather than zero — temperature.] The procedure outlined above is similar in spirit to the approach of classical thermodynamics rather than statistical mechanics.

It is rather intriguing that there exist analogues for the collapsing blackhole in the case of deSitter and even Rindler [31] . The analogue in the case of deSitter spacetime will be an FRW universe which behaves like a deSitter universe only at late times. (This is probably the actual state of our universe which has become dominated by a cosmological constant in the recent past). Mathematically, we only need to take $a(t)$ to be a function which has the asymptotic form $\exp(Ht)$ at late times. Such a spacetime is, in general, time asymmetric and one can choose a vacuum state at early times in such a way that thermal spectrum of particles exist at late times. Emboldened by the analogy with blackhole spacetime one can also directly construct quantum states (similar to Unruh vacuum of blackhole spacetime) which are time asymmetric, even in the exact deSitter spacetime, with the understanding that the deSitter universe came about at late times through a time asymmetric evolution.

The analogy also works for Rindler spacetime which is also time symmetric. The standard vacuum state respects this symmetry and we arrive at a situation in thermal equilibrium. The coordinate system for an observer with *time dependent* acceleration will, however, generalise the standard Rindler spacetime in a time dependent manner. In particular, one can have an observer who was inertial (or at rest) at early times and is uniformly accelerating at late times. In this case an event horizon forms at late times exactly in analogy with a collapsing blackhole. It is now possible to choose quantum states which are analogous to Unruh vacuum - which will correspond to an inertial vacuum state at early times and will appear as a thermal state at late times. The correspondence with CFT can now be used to compute $\langle T_{ab}\rangle$ in different 'vacuum' states to show [31] that radiative flux exists in the quantum states which are time asymmetric analogues of Unruh vacuum state.

But in deSitter or Rindler spacetimes there is *no* natural notion of energy (unlike in blackhole spacetimes which are asymptotically flat). In fact, it is not clear whether these spacetimes have an "energy source" analogous to the mass of the blackhole. While the deSitter spacetime is curved and one might consider the cosmological constant to change with evaporation, the Rindler spacetime is flat with (presumably) zero energy. Hence one is *forced* to interpret the quantum field theory in these spacetimes in terms of a state of thermal equilibrium with constant temperature but no radiation ("evaporation"). It is seems correct to conclude that the horizons *always* have temperature but it may not be conceptually straight forward to associate an entropy (or evaporation) with the horizon in all cases.

This might tempt one take the following point of view: In the case of blackholes, one considers the collapse scenario as "physical" and the natural quantum state is the Unruh vacuum. The notions of evaporation, entropy etc. then follow in a concrete

manner. The eternal blackhole (and the Hartle-Hawking vacuum state) is taken to be just mathematical constructs not realised in nature. In the case of Rindler, one may like to think of time symmetric vacuum state as natural treat the situation as one of thermal equilibrium. This forbids using quantum states with outgoing radiation which could make the Minkowski spacetime to radiate energy – which seems unlikely.

The real trouble arises for deSitter spacetime which is gaining in popularity. If the spacetime is asymptotically deSitter, should one interpret it as "evaporating" at late times with the cosmological constant changing with time ? This will make cosmological constant behave like quintessence models [31]. The energy source for expansion at early times (say, matter or radiation) is irrelevant just as the collapse details are irrelevant in the case of a blackhole. If this is the case, can one sensibly integrate the $dS = dE/T$ equation and obtain an entropy for deSitter spacetime, even though the spacetime is not asymptotically flat ? And finally, how does one reconcile the fact that the horizon in this case is observer dependent ? These issues are not analysed in adequate detail in the literature and are under study [31].

The discussion so far was based on the thermodynamic approach to entropy. One could ask whether it is possible to provide an alternative statistical mechanics interpretation of the entropy [via the equation $S = \ln g$] to complement the thermodynamical derivation and — if so — does it make a distinction between Schwarzschild, deSitter, and Rindler. The simplest case, of course will be that of a black hole. The study of the extensive literature currently available on this topic, with varying points of view, shows that we do *not* have a rigorous and unambiguous interpretation of the entropy of black hole in statistical mechanics terms *within the context of QFT in CST*. The situation is more unclear in the case of deSitter and Rindler. It is not even clear what are the degrees of freedom one is talking about though the best bet seems to be that they reside near the surface of the event horizon rather than inside or outside.

In the context of quantum gravity there have been attempts to relate S to underlying degrees of freedom of spacetime [15], [16], [17]. String theory provides an interpretation of S in a very special case of an extremal blackhole while the approach based on loop gravity leads to the proportionality between entropy and the horizon area in a general context though it cannot provide the proportionality constant unambiguously. In both these quantum gravitational approaches, certain degrees of freedom are identified and the logarithm of these degrees of freedom leads to an entropy. These quantum gravitational approaches, unfortunately, are not of much help in comparing deSitter and Schwarzschild spacetimes. String theory has difficulty in accommodating a positive definite cosmological constant [20] and — in any case — the formalism cannot even handle normal Schwarzschild blackhole rigorously at present. The loop gravity approach suffers from the difficulty that — while it attributes an entropy to *any* horizon — the derivation is kinematical in the sense that there could be selection rules in the theory which have a bearing on emission of radiation. Until these are incorporated, it is not possible to proceed from the entropy to temperature. In short, QG models obtain an entropy but not temperature while QFT in CST can lead to temperature but not to entropy in a straightforward manner.

There is another intriguing connection between trapped surfaces and quantum gravity. I have given detailed arguments elsewhere [18] to show that the event horizon of a Schwarzschild blackhole acts as a magnifying glass, allowing us to probe Planck scale physics. Consider, for example, a physical system described by a low energy Hamiltonian, $H_{\rm low}$. By constructing a blackhole made from the system with this

Hamiltonian and requiring that the blackhole should have a density of states that is immune to the details of the matter of which it is made, one can show that the Hamiltonian, $H_{\rm true}$ describing the interactions of the system at transplanckian energies must be related to $H_{\rm low}$ by $H^2_{\rm true} = \alpha E_P^2 \ln[1 + (H^2_{\rm low}/\alpha E_p^2)]$ where α is a numerical factor. Of course, the description at transplanckian energies cannot be in terms of the original variables in the rigorous theory. The above formula should be interpreted as giving the mapping between an effective field theory (described by $H_{\rm true}$) and a conventional low energy theory (described by $H_{\rm low}$) such that the blackhole entropy will be reproduced correctly.

In fact, one can do better and construct a whole class of effective field theories [18] such that the one-particle excitations of these theories possess the same density of states as a Schwarzschild blackhole. All such effective field theories are non local in character and possess a universal two-point function at small scales. The nonlocality appears as a smearing of the fields over regions of the order of Planck length thereby confirming ones intuition about microscopic structures, trapped surfaces and blackhole entropy.

7. Cosmological constant – to be or not to be

The last theme I want to mention is the issue of cosmological constant which is the deepest question that confronts any attempt to combine the principles of general relativity and quantum theory. If the current observational evidence – suggesting the existing of a small but nonzero cosmological constant – does not go away, theoreticians have a serious problem in their hands. Let me briefly review the difficulties and possibilities.

To begin with, cosmological constant is *not* a problem in classical general relativity. Classical physics has constants which stay as constants and one is allowed to give any value to them. If we write the Einstein's equations as $G_{ik} + \Lambda g_{ik} = 8\pi G_N T_{ik}$ we are free to choose any value we like for the two constants (G_N, Λ). Further, one cannot construct any quantity with dimension of Λ from G_N and c alone so there is no question of fine tuning.

The situation changes in three respects when one brings in quantum theory. (i) With $(G_N, c, \hbar)$ one can construct a dimensionless combination ΛL_P^2 and one may be justifiably curious why this quantity is somewhat small — being less than 10^{-120}. (ii) The coupling constants in quantum theory "run". In any sensible model, with a UV cutoff around Planck scale, the value of ΛL_P^2 will run to a number of order unity; that is, a tiny value is unnatural in the technical sense of the term. (iii) Any finite vacuum energy density V_0, including the constants added to potential energy terms of scalar fields, say, will contribute a term $(-V_0 g_{ik})$ at the right hand side of Einstein's equations. This is mathematically indistinguishable from the cosmological constant. It is not clear why $V_{0,net} L_P^2$ is less than 10^{-120}. The situation is aggravated by the fact that we do not know of any symmetry which requires $V_{0,net} L_P^2$ to be zero.

Ever since recent cosmological observations suggested the existence of a nonzero cosmological constant, there has been a flurry of theoretical activity to " explain " it, none of which even gets to the first base. One class of models invokes some version of anthropic principle; but since anthropic principle never predicted anything, I do not consider it part of scientific methodology. The second class of models use a scalar field with an "appropriate" potential $V(\phi)$ to "explain" the observations. These models are all trivial and have no predictive power because it is always possible to choose a $V(\phi)$

to account for any sensible dynamical evolution of the universe. Since the triviality of these models (which are variously called "quintessence", "dark energy" ....) does not seem to have been adequately emphasised in literature, let me briefly comment on this issue [32].

Consider any model for the universe with a *given* $a(t)$ and some known forms of energy density $\rho_{\rm known}(t)$ (made of radiation, matter etc) both of which are observationally determined. It can happen that this pair does not satisfy the Friedmann equation for an $\Omega = 1$ model. To be specific, let us assume $\rho_{\rm known} < \rho_c$ which is substantially the situation in cosmology today. If we now want to make a consistent model of cosmology with $\Omega = 1$, say, we can invoke a scalar field with the potential $V(\phi)$. It is trivial to choose $V(\phi)$ such that we can account for *any* sensible pair $[a(t), \rho_{\rm known}(t)]$ along the following lines: Using the given $a(t)$, we define two quantities $H(t) = (\dot{a}/a)$ and $Q(t) \equiv 8\pi G\rho_{\rm known}(t)/3H^2(t)$. The required $V(\phi)$ is given parametrically by the equations:

$$V(t) = (1/16\pi G)H(1-Q)\left[6H + (2\dot{H}/H) - (\dot{Q}/1-Q)\right] \tag{19}$$

$$\phi(t) = \int dt\, [H(1-Q)/8\pi G]^{1/2}\left[\dot{Q}/(1-Q) - (2\dot{H}/H)\right]^{1/2} \tag{20}$$

All the potentials invoked in the literature are special cases of this formula [32]. This result shows that *irrespective of what the future observations reveal about $a(t)$ and $\rho_{\rm known}(t)$* one can always find a scalar field which will "explain" the observations. Hence this approach has no predictive power. What is worse, most of the $V(\phi)$ suggested in the literature have no sound particle physics basis and — in fact — the quantum field theory for these potentials are very badly behaved on nonexistent.

It is worth realising that the existence of a non zero cosmological constant will be a statement of fundamental significance and constitutes a conceptual contribution of cosmology to quantum gravity. The tendency of some cosmologists to treat Ω_Λ as one among a set of, say, 17 parameters [like $\Omega_{rad}, \Omega_B, n, ...$] which need to be fixed by observations, completely misses the point. Cosmological constant is special and its importance transcedends cosmology.

At present we do not have a fundamental understanding of cosmological constant from any approaches to quantum gravity. There are no nontrivial string theoretical models incorporating $\rho_V > 0$; loop gravity can incorporate it but does not throw any light on its value. It should be stressed that the nonzero value for $\rho_V \neq 0$ does *not* imply deSitter (or even asymptotically deSitter) spacetime. Hence the formalism should be capable of handling ρ_V without deSitter geometry.

To give an example of a more fundamental way of thinking about cosmological constant, let me describe an idea in which cosmological constant is connected with the microstructure of spacetime. In this model we start with $\Lambda = 0$ but generate a small value for this parameter from two key ingredients: (i) discrete spacetime structure at Planck length and (ii) quantum gravitational uncertainty principle. To do this, we first note that cosmological constant can be thought of as a lagrange multiplier for proper volume of spacetime in the action functional for gravity:

$$A_{grav} = \frac{1}{2L_P^2}\int d^4x R\sqrt{-g} - \frac{\Lambda}{L_P^2}\int d^4x\sqrt{-g}; \tag{21}$$

In any quantum cosmological models which leads to large volumes for the universe, phase of the wave function will pick up a factor of the form $\Psi \propto \exp(-i(\Lambda/L_P^2)\mathcal{V})$,

where $\mathcal{V}$ is the four volume, from the second term in (21). Treating $(\Lambda/L_P^2, \mathcal{V})$ as conjugate variables (q,p), we can invoke the standard uncertainty principle to predict $\Delta\Lambda \approx L_P^2/\Delta\mathcal{V}$. Now we make the crucial assumption regarding the microscopic structure of the spacetime: Assume that there is a zero point length of the order of L_P so that the volume of the universe is made of several cells, each of volume L_P^4. Then $\mathcal{V} = NL_P^4$, implying a Poisson fluctuation $\Delta\mathcal{V} \approx \sqrt{\mathcal{V}} L_P^2$. and leading to

$$\Delta\Lambda = \frac{L_P^2}{\Delta\mathcal{V}} = \frac{1}{\sqrt{\mathcal{V}}} \approx H_0^2 \tag{22}$$

which is exactly what cosmological observations imply! Planck length cutoff (UV limit) and volume of the universe (IR limit) combine to give the correct $\Delta\Lambda$. Of course, this makes Λ a stochastic variable and one needs to solve Friedmann equations using a stochastic source [31].

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